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A TEMPERATURE PROBE FOR THE DETERMINATION OF HEAT TRANSFER AT A WALL IN TURBULENT FLOW

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Abstract—An original technique is proposed for the determination of the wall heat flux in turbulent air flow. The technique is based upon the inner law correlations for velocity and temperature, and consists of placing a thermocouple at a specified location in the inner law region. The measured temperature difference between the probe and the wall is related to the heat flux at the wall. Application is demonstrated by the calibration of two types of heat flux probes, with the accuracy of calibrations being to ± 2 per cent. Small modifications to the calibrations are expected for high temperature and high Prandtl number flows.

NOMENCLATURE

c_p, specific heat at constant pressure [Btu/lb-°F];
d, diameter of a probe [ft];
g, gravitational acceleration [ft/s^2];
k, thermal conductivity [Btu/h-ft °F];
l, length of unsheated thermocouple leads at the junction [ft];
Pr, Prandtl number [dimensionless];
q, heat flux [Btu/h-ft^2];
t, temperature [°F];
t^+, inner law temperature [dimensionless];
u, velocity [ft/s];
u^+, inner law velocity [dimensionless];
u_*, wall shear velocity [ft/s], $u_* = (\sqrt{\tau_0/\rho_0})$;
y, distance from a wall [ft];
y^+, inner law dimensionless distance from a wall [dimensionless].

Greek symbols
β, inner law heat flux parameter,
$$\beta = \frac{q_0\,(\sqrt{\tau_0/\rho_0})}{c_{p0}g\tau_0 t_0} \text{ [dimensionless]};$$
μ, dynamic viscosity [lb-s/ft^2];
ν, kinematic viscosity [ft^2/s];
ρ, fluid density [lb-s^2/ft^4];
τ, shear stress, $\tau = \mu \frac{du}{dy}$ [lb/ft^2];
φ, velocity inner law function [dimensionless];
φ_1, temperature inner law function [dimensionless];
φ^1, Preston tube calibration [dimensionless];
φ_1^1, heat flux probe calibration [dimensionless].

Subscripts
i, incident streamline with a probe;
m, measured value of velocity or temperature;
0, wall value;
y, value of velocity or temperature at y.

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1. INTRODUCTION

DIRECT measurement of the rate of heat transfer from a wall to a flowing fluid is a difficult operation wherever the heat transfer is not uniformly distributed. If for geometrical reasons the rate is the same at all points on the surface, then a thermal balance provides the heat-transfer rate. However in most flow situations heat is transferred from a solid surface at a variable rate, dependent upon the convective properties of the flow. Thus a point by point measurement is required. In theory this information can be obtained by measuring the surface temperature gradient, either in the wall or in the fluid. However the requirements of a good heat-transfer material include that of high thermal conductivity, with the result that a small wall temperature gradient has to be evaluated. Moreover the usual methods of obtaining temperature gradients restrict the technique to a limited number of points. A method can be based upon the surface temperature gradient in the fluid, the gradient is large enough to be measured, but is limited to the viscous sublayer of the flow. The thickness of this layer is usually so small that the thermocouple junctions of even the smallest probes are of the same order of magnitude. Thus spatial resolution of the surface temperature gradient in the fluid is usually too difficult to be achieved with reliability or accuracy.

A superior technique is proposed and will be demonstrated by actual heat-transfer measurements. It is based on the assumption that the inner law velocity and temperature correlations are related to the local surface flux of heat. The correlations are known to exist for turbulent heat transfer from a smooth surface for moderate variations of temperature, and with approximate corrections for fluid properties the correlations apply to large temperature variations as well [1].

The technique consists of placing a thermocouple of suitable geometry in the inner law portion of the turbulent flow. The temperature difference from the wall value is a function of the variables which control the inner law temperature correlation, with an additional factor accounting for the probe geometry. For a calibrated probe the temperature difference can thus be directly related to the heat-transfer rate at the wall. The technique is analogous to the determination of the wall shear stress by a Preston tube, since there is ample evidence to indicate that its calibration is achieved by the velocity inner law parameters [2, 3].

The velocity inner law correlation is determined by the wall shear stress, kinematic viscosity in the wall region, and the distance from the wall, [1]. The correlation can be expressed as

$$u^+ = \frac{u_y}{u_*} = \varphi\left(\frac{u_* y}{\nu}\right) = \varphi(y^+). \tag{1}$$

The temperature correlation can be expressed as

$$t^+ = \frac{t_y - t_0}{q}\rho q c_p u_* = \left[\left(\frac{u_* y}{\nu}\right), \left(\frac{c_p \mu}{k}\right)\right] = \varphi_1(y^+, Pr). \tag{2}$$

Since the inner law correlations of velocity and temperature are valid close to a surface and are controlled by the local wall shear stress and heat flux, it is evident that these correlations will depend upon the local wall parameters regardless of their lateral or longitudinal variations. This is a fortunate circumstance since the inverse applies, permitting us to obtain the local wall parameters with probes placed in the inner law region. The region is free of large velocity or temperature gradients, but contains suitable large velocity and temperature differences, with respect to the wall, to permit high accuracy resolutions of the wall shear stress and heat flux.

2. DESIGN OF SHEAR STRESS AND HEAT FLUX PROBES

Preston and others [2, 3] have established a calibration for stagnation tubes placed directly on the wall. The tubes had a significant diameter, the top extended well beyond the viscous sub-layer into the inner law logarithmic region, the edge of which is nominally $y^+ = 30$. A significant flow disturbance was produced by each tube due to the size. However it was noted that the inner law parameters controlled the flow disturbances so that a single calibration curve was obtained for probes with a given geometry, which was specified by Preston. The calibration was given as

$$\frac{u_*}{u_m} = \varphi^1 \frac{u_m d}{2\nu}. \quad (3)$$

In view of the success of the Preston tube technique in predicting local wall shear stress, it is logical to infer that an analogous technique can be developed for the determination of the wall heat flux. The calibration of a heat flux therefore requires the specification of the wall shear stress, the wall heat flux, the measured temperature difference between the probe and the wall, and finally the probe geometry. Since the wall shear stress must be determined by a Preston tube, it is desirable to present the probe calibration in the form of the temperature inner law as

$$t_m^+ = \frac{t_m - t_0}{q} \rho g c_p u_* = \varphi_1^1 \left[\left(\frac{u_* d}{\nu}\right)\left(\frac{c_p \mu}{k}\right)\right]. \quad (4)$$

3. DESCRIPTION OF EXPERIMENTAL EQUIPMENT AND TECHNIQUES

The calibration of the heat flux probes required a controlled range of heat-transfer and turbulent flow rates. These requirements were provided in two heat-transfer rigs, the first a heated flat plate parallel to the flow in a wind tunnel, the second a heated vertical square duct.

3.1. *Heated aluminum plate*

An aluminum plate 2 by 6 ft and 1-in thick was employed. Chromel "C" heater tape 0·5 by 0·005-in thick was positioned in 0·5-in slots machined into the back of the plate. The heater tape was insulated by fiberglass and high temperature varnish. The slots were positioned at 1-in intervals to provide three circuits of heater tape of 50-ft length each, with a resistance of 0·2040 Ω per ft. External rheostats and calibrated ammeters were used to determine the rate of electrical power dissipation. Power was obtained from an independent d.c. generator driven by a three-phase 550 V 15 hp motor. After the initial period of warm-up no variations in the supply voltage were detected.

The surface temperature of the plate was determined from twenty-one thermocouples which were placed in wells drilled to within 0·1 in of the surface of the plate. The thermocouples were fabricated from twenty-four gauge copper-constantan wires, with the leads positioned along the plate underside for several inches to overcome conductive errors. The moderate heat-transfer rates required no correction for temperature gradients through the plate.

The reported tests were conducted in a

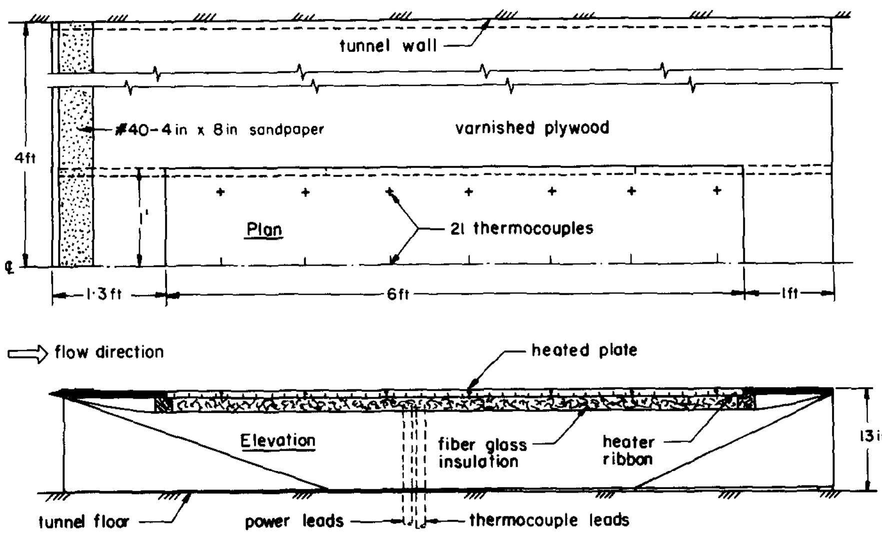

FIG. 1. Flat plate detail.

non-recirculating wind tunnel for velocities from 10 to 24·5 ft/s. The working section of the tunnel was 8-ft wide by 4-ft high. Uniform velocity profiles were obtained outside of the wind tunnel boundary layers by a carefully designed inlet section consisting of a honeycomb and three sets of screens. The heated plate was tested in the central region of the tunnel which was free of boundary-layer influence, by positioning the plate in a plywood deck (Fig. 1). The deck and plate were positioned approximately 13 in above the wind tunnel floor, with the back of the structure shimmed approximately $\frac{1}{2}$ in to provide an undisturbed potential flow upstream of the leading edge. The underside of the structure was carefully streamlined to reduce blockage while providing a 2-in layer of fiberglass insulation on the back of the plate. To stabilize the boundary layer a 4-in strip of No. 40 sandpaper was incorporated on the upper surface near the leading edge.

The heater circuits were adjusted to give a constant surface temperature for the range of tests. The calculated convection rates were based upon the rate of electrical dissipation with corrections for conductive and radiation losses. The temperature and velocity profiles were determined 5 ft from the leading edge of the aluminum plate. Lateral explorations at this position indicated that the temperature and velocity profiles were not distorted over the central 12 in of plate. All measurements of profiles were made well within this region.

3.2. *Heated square aluminum duct*

A square duct was fabricated from architectural grade extruded aluminum, with inside dimensions of 3·767 by 3·767 in, with sharp inside corners, and with a wall thickness of 0·188 in. Two 15-ft lengths were bolted together to give a test section length of 30 ft, or ninety-six hydraulic diameters. The duct was mounted vertically to eliminate velocity and temperature distortions at the moderate Reynolds numbers achieved in the test programme. Thermal expansion was accommodated by suspending the assembly from the top and by providing an extensible rubber sleeve between the fan and settling chamber (Fig. 2).

The duct was heated by ten circuits of 0·5 by 0·005-in thick by 50-ft long Chromel "C" heater tape. For the heat rates employed the heater tapes were connected together as pairs in series to external rheostats which were connected to the above mentioned motor-generator set. The aluminum duct was insulated by a layer of fiberglass cloth and high temperature varnish before the heater tapes were spirally wrapped onto the duct.

A guard heater of 0·125 by 0·005-in thick Chromel "A" heater tape was wrapped on the final 4 in of the duct to compensate for end losses. The guard heater was independently controlled by a rheostat.

The heater tapes were covered by a layer of fiberglass cloth and by two layers of 2-in thick foamglass insulation.

The heater circuits were adjusted to give a constant temperature drop from the walls of the duct to the centerline over the 20–30 hydraulic diameters of established flow before the exit. Thus the longitudinal temperature gradients along the walls and along the centerline were identical as determined by wall and centerline thermocouples positioned along the fifteen hydraulic diameters.

Heat-transfer rates were determined from the rate of temperature increase of the air in the longitudinal direction and the discharge of air. An independent heat balance based upon the rate of electrical dissipation and calculated heat loss gave very good agreement. Discharge was determined by a calibrated flow meter, with variations in the rate being controlled by a damper positioned at the fan outlet.

3.3. *Shear stress measurements*

The wall shear stress was determined for the flat plate and the square duct by calibrated Preston tubes. In the case of the square duct the data of Leutheusser [4] were used.

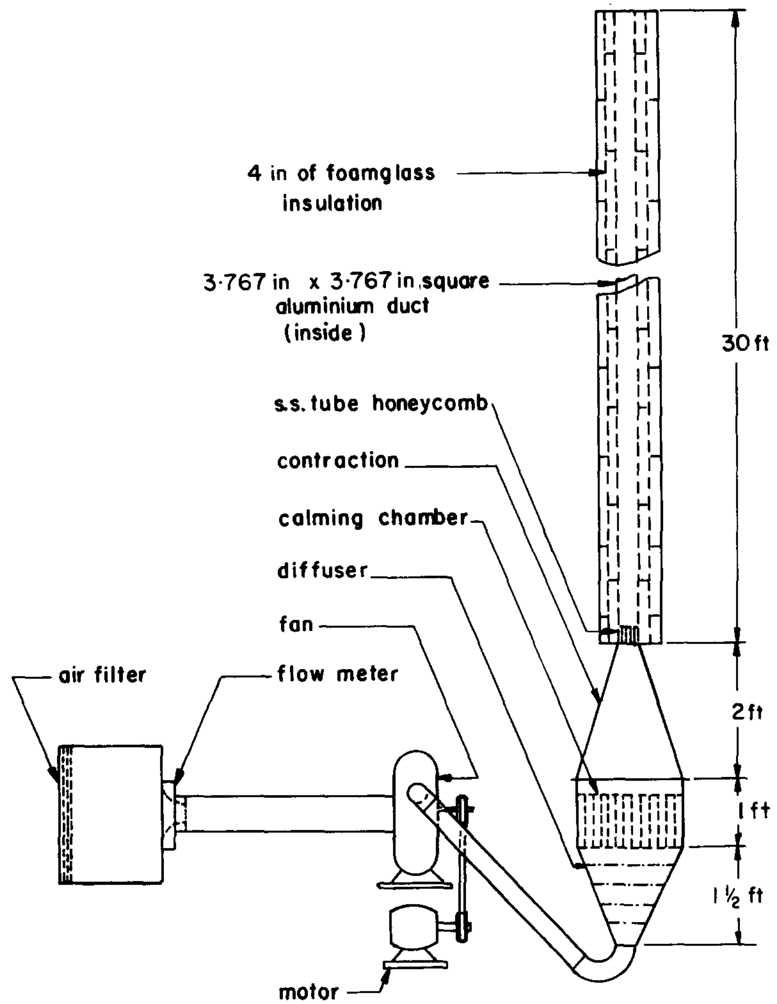

FIG. 2. Square duct detail.

3.4. *Temperature measurement*

Temperature readings were obtained with a Photovolt recorder. The recorder had a full scale deflection of 0·5 mV for 10 in of chart, with a full scale response of 0·75 s. Highest accuracy was obtained by recording the temperature difference between the heat flux probe and the wall directly, rather than recording absolute values which then were subtracted.

4. HEAT FLUX PROBE DETAILS AND CALIBRATION

Two heat flux probes were developed and calibrated. The first, type A, was designed to simulate the geometry of a Preston tube by placing a large thermocouple bead just in front of a square ended tube, Fig. 3. The probe was calibrated in the vertical square duct by employing average wall heat flux and shear stress parameters. Temperature readings obtained with the probe in direct contact with a wall were dependent upon the probe contact pressure, and thus were subject to experimental error. A satisfactory arrangement which lead to highly accurate calibrations was achieved by separating the probe from the wall by a small distance. The best results were obtained when the separation distance was equal to the thickness of the viscous sublayer, then a ± 20 per cent variation of the separation distance lead to only a ± 2 per cent variation of the measured temperature. For these experiments the required accuracy of positioning the probe was $\pm 0{\cdot}001$ in.

The calibration of the type A probe is given in Fig. 4. The calibration of a Preston tube is included with the measured velocity given in terms of the inner law velocity parameters. Such a correlation is not convenient for determining the wall shear stress, but permits us to compare the response to that of the heat flux probe. It is noted that the heat flux probe has a lower response than that of the Preston tube, when t_m^+ and u_m^+ are compared, to a large extent this is due to the Prandtl number of air which is 0·72. The fluid must have a Prandtl number of one if temperature and velocity profiles are identical. Thus the magnitude and

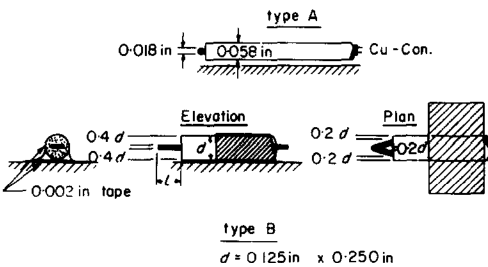

FIG. 3. Heat flux probe detail.

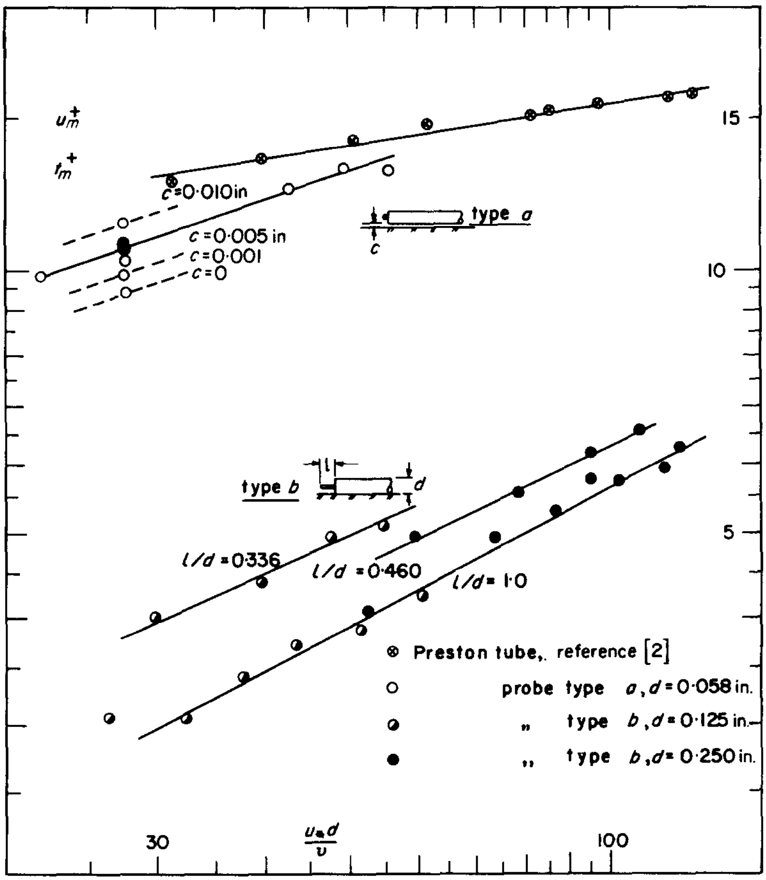

FIG. 4. Calibration of Preston and heat flux probes.

accuracy of the heat flux probe type A calibration is felt to be excellent.

A second series of probes, type B, were developed from commercial thermocouple wires assembled in a stainless steel sheath with an inert packing powder. These thermocouple assemblies provide reproducible probe geometries when fabricated as shown in Fig. 3. Furthermore it was possible to construct geometrically similar probes from different sizes of the commercial stock.

Two type B probes were calibrated (Fig. 4), by using the heated plate. The probes were first calibrated for $l/d = 1{\cdot}0$, then for reduced values by reforming the thermocouple junctions. Since the calibrations of these probes were much lower than those of the type A probe or of the Preston tube, a discussion is required of the two factors influencing the results. First, heat was transferred from the plate to the type B probes by conduction. This effect was partially eliminated by placing a 0·002-in layer of plastic backed tape on the plate surface before the probe was taped to the plate (Fig. 3). However the resulting conduction of heat to the probes still resulted in a reduction of the measured temperature difference, and hence a reduction of t_m^+. Fortunately the heat balance of the probes was controlled by the inner law flow region in which they were immersed. Thus the conduction effect was calibrated by the inner law parameters, and was considered as a part of the geometrical calibration factor. The second factor was related to the position of the thermocouple junction, since t_m^+ increased as l/d was decreased. This effect was due to the so called Pitot tube shift effect, by which streamlines were shifted towards lower velocity regions by a blockage placed in flows with velocity gradients. The effect was primarily hydrodynamic, since the stagnation pressure distribution on the upstream face of the blockage decreased in the direction of decreasing velocities, creating a component of flow which was down the velocity gradient, thus higher velocity streamlines were drawn toward the position of the blockage. As the blockage effect increased, that is as l/d was decreased, t_m^+ was increased. This second effect was also controlled by the inner law parameters since the flow distortion depended on the velocity gradient which is a function of inner law parameters.

These qualitative explanations indicate that the performance of the B type probes is satisfactory. It is noteworthy that geometrically similar probes of different sizes have a single calibration function as required by the velocity and temperature inner law parameters. The accuracy of the calibrations appear to be as good as that of the A type probe, that is to ± 2 per cent.

5. ADDITIONAL FACTORS AFFECTING THE CALIBRATION OF A HEAT FLUX PROBE

The present results were obtained for moderate temperature differences of 25 degF. Variations in fluid properties and thermal radiation effects are therefore negligible. Such is not the case for very large heat-transfer rates, where corrections to the velocity and temperature inner laws are required as well as corrections for thermal radiation between the probe and wall. The techniques required to compensate for these effects appear to be straightforward.

Inner law velocity and temperature correction for large temperature variations throughout the fluid, for a fluid with a Prandtl number of air, can be obtained from the calculations of Deissler [1], in which the temperature inner law is expressed as a function of β, where

$$\beta = \frac{q_0 \sqrt{(\tau_0/\rho_0)}}{c_{p0} g \tau_0 y_0}. \tag{5}$$

The interesting feature of the modified inner laws is that very small corrections are encountered for large temperature differences t_m/t_0 if y^+ is less than 100. With reference to Deissler's findings a ratio of t_m/t_0 of 1·4 resulted in a β value of 0·015. Now if y^+ is selected as sixty, which is approximately the value for the type A probe, then t^+ is reduced by less than one unit, or by approximately 10 per cent, for heating. A similar but opposite effect occurs for cooling.

Thermal radiation corrections for heat transfer between the probe and wall can be obtained from existing data for thermocouples [5]. Again it is interesting to note that the corrections are usually moderate until the wall and probe temperatures approach 800°F. A standard correction for the present probes can then be introduced, or better still a shielded probe can be substituted.

Finally, the present results are for air flow only. Heat transfer from fluids with Prandtl numbers other than 0·72 require further calibration. It is noted, however, that the inner law temperature curve is modified in a known manner. Thus the calibration of the heat flux probe can be estimated by noting the t_m^+ response for air flow. This response can be interpreted as due to the intercept of a particular streamline of the inner law with the probe. Let us define the undisturbed position of this streamline as y_i^+. Usually this streamline is further from the wall than the probe centerline due to the Pitot tube shift effect. We then enquire how the temperature of the y_i^+ streamline is altered by variations of the Prandtl number. Now assuming that the probe continues to intercept the streamline at y_i^+ for various Prandtl numbers, we obtain the desired estimate of the dependence of the heat flux probe calibration upon the Prandtl number. This correction appears to be moderate for Prandtl numbers in the range of 0·5–10.

6. APPLICATION OF A HEAT FLUX PROBE

The successful application of a heat flux probe requires careful consideration of the calibrations shown in Fig. 4. It is obvious that the probe in the form described herein, requires calibration, and that the measurements taken must lie within the calibrated range. In practice the wall shear stress, must be determined and since this must be done with accuracy, a Preston tube is usually used. Then with the wall shear stress and hence u_* determined at all points of interest, one or more heat flux probes are selected such that u_*d/ν is within the calibrated range. With the value of t_m^+ now known for every point of interest because u_* is known, only the measured temperature difference between the probe and the wall is required to determine the local heat flux. This can be seen from equation (4) since

$$q = \frac{t_m - t_0}{t_m^+} \rho g c_p u_*. \tag{6}$$

7. CONCLUSIONS

The direct measurement of the rate of heat transfer from a wall to a flowing fluid is recognized to be difficult. Indeed the theoretical procedure of determining the temperature gradient at the surface of a heat-transfer wall, either in the metal or in the fluid is shown to be impractical. Generally the temperature gradients within the metal are too small while the useful temperature gradients within the fluid are restricted to too small a region at the wall.

A new technique is developed and demonstrated by calibrating two types of heat flux probes. The method is highly accurate and is based upon the inner law correlations for velocity and temperature. For air flow it is known that these laws are solely dependent upon the wall shear stress, wall heat flux and local kinematic viscosity. Thus for a calibrated temperature probe the measured difference in temperature between the probe and wall can be related to the local heat flux at the wall. It is noted that the calibration can be reproduced to an accuracy of ±2 per cent, and that the extension of the calibration to high temperatures and other fluids is straight forward.

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Résumé—Une technique originale est proposée pour la détermination de flux de chaleur à la paroi dans un écoulement turbulent d'air. Cette technique est basée sur les relations intérieures pour la vitesse et la température et consiste à placer un thermocouple à un endroit spécifique de la région de la loi intérieure. La différence de température mesurée entre la sonde et la paroi est liée au flux de chaleur pariétal. L'étalonnage de deux types de sondes de flux de chaleur en donne une application, la précision des étalonnages étant de $\pm 2\%$.

On s'attend à de faibles modifications des étalonnages pour des écoulements à température élevée et à grands nombres de Prandtl.

Zusammenfassung—Zur Bestimmung des Wärmetransportes an der Wand bei turbulenter Luftströmung wird eine neuartige Technik vorgeschlagen. Die Technik beruht auf Korrelationen für Geschwindigkeit und Temperatur, wofür ein Thermoelement an einer bestimmten Stelle angebracht werden muss. Die gemessene Temperaturdifferenz zwischen der Messstelle und der Wand wird in Beziehvng gesetzt zum Wärmestrom an der Wand. Eine Anwendung ist gezeigt in der Kalibrierung zweier Arten von Wärmeflussmessern. Die Genauigkeit dabei beträgt ± 2 Prozent. Kleine Änderungen der Kalibrierung sind für hohe Temperaturen und Strömungen grosser Prandtl-Zahl zu erwarten.

Аннотация—Предложен оригинальный метод определения теплового потока на стенке в турбулентном течении воздуха. Метод основан на корреляциях скорости и температуры для внутренней задачи и состоит в том, что термопара располагается в определенном положении в области действия этих корреляций. Замеренная разность температур зонда и стенки связана с тепловым потоком на стенке. Применение метода проиллюстрировано на примере градуировки двух типов зонда с точностью до $\pm 2\%$.

Предполагается, что при высоких температурах и больших числах Прандтля градуировка должна быть несколько модифицирована.